**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans= B

From a standard normal table, we can find that the probability P(Z > 0.625) is approximately 0.2676. So the answer is 0.2676 (Option B)

It means that there is a 26.76% chance that the service manager cannot meet his commitment.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans=

False.

We can calculate the proportion of employees who are older than 44 and between 38 and 44 by using the standard normal table or calculator.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans=

False

The statement is incorrect because we cannot determine the exact number of employees who would be attracted to a training program based on the mean and standard deviation of the current ages of the employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans=

2X1 and X1 + X2 are both linear combinations of iid normal random variables X1 and X2.

2X1 is a random variable that is equal to twice the value of X1. The distribution of 2X1 is also normal with mean 2μ and variance 4σ^2.

X1 + X2 is a random variable that is equal to the sum of the values of X1 and X2. Since X1 and X2 are iid normal random variables with the same mean and variance, the sum of X1 and X2 is also a normal random variable with mean 2μ and variance 2σ^2.

So, the distribution of 2X1 is different from the distribution of X1 + X2, the first one is N(2μ, 4σ^2) and the second one is N(2μ, 2σ^2)

In summary, the difference between 2X1 and X1 + X2 is in their variances, 2X1 has a variance of 4σ^2 and X1 + X2 has a variance of 2σ^2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans= E

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans=

the rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company is (295.15, 784.35)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans= the answer is Rs. 463.57

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans=

the probability that the annual profit for the first division is less than 0 is 0.17

the probability that the annual profit for the second division is less than 0 is 0.13

As the probability of making a loss is larger for the first division, it has a larger probability of making a loss in a given year.